The Second Derivative Test and Optimization

Math 130 - Essentials of Calculus

11 November 2019

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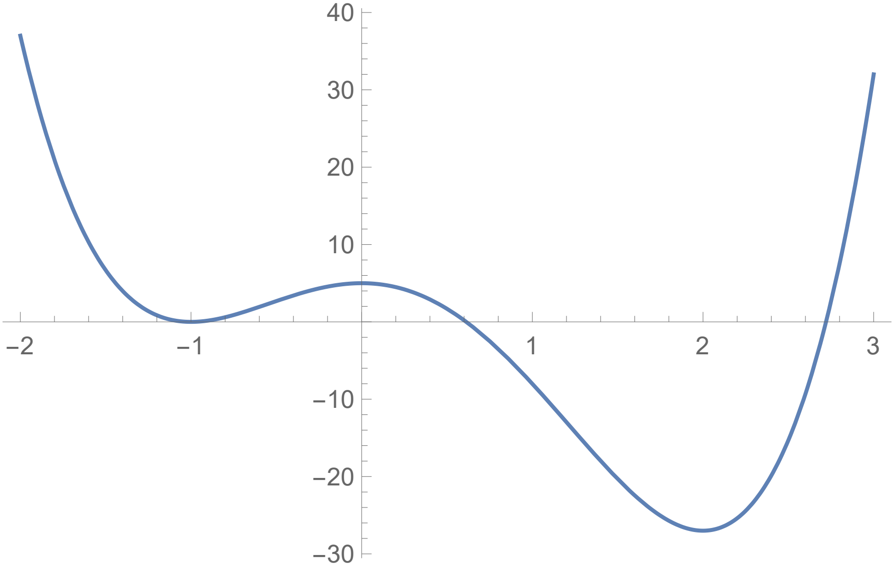
Observe the location of the critical numbers in the chart for f''...



40
30
20
10
−10
−20
−30
−2
−1
1
2
3

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Example

Find the local maximum and minimum values of $y = x^4 - 4x^3$.

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For the function $y = 3x^5 - 5x^3 + 3$

1. *Find the local maximum and miminum values of* f.
2. *Find the intervals of concavity and the inflection points.*

Starting Example

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A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?

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4. *Determine the feasible domain of the primary equation. That is, determine the values for which the stated problem makes sense.*
5. *Determine the desired maximum or minimum value using calculus.*

Now You Try It!

Example

Find the dimensions of a rectangle with perimeter $100m$ *whose area is as large as possible.*